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P. Hajela and G. Dvorak

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Rensselaer Polytechnic Institute
110 8th street
Troy, NY 12180-3590

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14. ABSTRACT
The final stage of the proposed research was completed during this performance period. The focus of this work was on integrating the composite damage analysis programs into a framework where they could be effectively used for design optimization. Critical to this phase of the work was a problem formulation that took into consideration the multiscale nature of the damage process, and fully accounted for the residual life in the composite structure in a post-failure (local failure mode). A system effectiveness approach was developed as a logical framework for the design problem formulation. This approach fully accounts for the probabilities of going from one state of structural damage to another. The problem formulation also allows for creating objective criteria that include system reliability and residual life in the structure.
The approach was demonstrated using simple composite design problems.

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Performance Report
FA 9550-05-1-0140

Uncertainty Modeling in the Analysis and Design of Damage Tolerant Composite Structural Systems

By

Prof. P. Hajela, Prof. G. Dvorak

Mechanical, Aerospace and Nuclear Engineering
Rensselaer Polytechnic Institute
Troy, NY 12180

Submitted to

Dr. Victor Giurgiutiu
Air Force Office of Scientific Research

Summary Statement

This document summarizes some of the key finds of the AFOSR research project. The report is divided into four sections, each addressing the major new contributions that resulted from the work

a. Section I describes the progress in the development of the transformation field analysis (TFA) based progressive damage model, referred to as the micro-TFA model, for laminated composites.
b. Section II describes the uncertainty propagation across different scales of the model, and how it can be used to predict degraded performance of the composite with a defined probability. It also illustrates the need for specialized techniques to handle uncertainty in predicting reliability, as well-known distributions of uncertainty in basic material properties may yield quite varied distributions in composite strength.
c. Section III addresses one issue of incorporating this analysis model in a design optimization environment – that of the high cost of analysis and the need to create surrogate representations of the exact damage progression analysis. A new approach referred to as a Local Matamodeling-Based Uncertainty Characterization (LMUC) for reducing the computational cost of performing Monte Carlo simulations on the micro-TFA model is proposed.
d. In Section IV, a design problem formulation based on a state transition approach is introduced, which allows for the handling of multiple failure modes in a rational manner. This methodology termed as a system effectiveness approach, models designer preference as to acceptability of degraded performance, and is used to develop optimal designs. A comparison of these designs against those obtained from a more widely used competing risk methodology provides insight into the advantages of the new approach.

Personnel

The following personnel were involved in the project at various stages of the program.

Drs. P. Hajela and G. Dvorak, Professors, MANE Department, Rensselaer Polytechnic Institute

Dr. Yehia Bahei El-Din, Visiting Professor, MANE Department, Rensselaer Polytechnic Institute

Dr. Anoop Mullur, Post-Doctoral Research Associate, MANE Department, Rensselaer Polytechnic Institute (now with Exxon Research, Houston, TX)

Dr. Ritesh Khire, Post-Doctoral Research Associate, MANE Department, Rensselaer Polytechnic Institute (now with United Technologies Research Center, Hartford, CT)

Mr. Varun Sakalkar, Doctoral Student, MANE Department, Rensselaer Polytechnic Institute (May 15, 2007 – present; expected graduation is February 2010)

Publications

The following major publications resulted from the work, two are pending journal reviews and possible publication in the archival literature.

A. Mullur, P. Hajela, and Yehia Bahei-El-Din "Uncertainty Management in Design Optimization of Coupled Systems," Proceedings of the 11th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Portsmouth, Virginia, September 2006.

P. Hajela and A. Mullur, "Modeling and Management of Uncertainties in Multidisciplinary Design Optimization," Plenary Lecture, 2nd LNCC Meeting on Computational Modeling, Petropolis, Brazil, August 2006.

P. Hajela and A. Mullur, "Uncertainty Modeling in Structural Design: Recent Developments," P. Hajela and A. A. Mullur, Innovation in Computational Structures Technology, eds. B.H.V. Topping, G. Montero, and R. Montenegro, pp. 51-74, Saxe-Coburg Publications, Kippen, UK, 2006.

P. Hajela and S. Vittal, "Optimal Design in the Presence of Modeling Uncertainties," ASCE Journal of Aerospace Engineering, pp. 204-216, Vol. 19, No. 4, October 2006.

A. Mullur, Y. Bahei-El-Din, P. Hajela, J. Peters and G. Dvorak, "Nondeterministic Modeling of Progressive Failure in Laminated Composites", proceedings of the 47th AIAA/ASME/ASCE/AHS/ASC SDM Conference, Newport, Rhode Island, May 2006.

R. Khire, P. Hajela, and Yehia Bahei-El Din, "Handling Uncertainty Propagation in Laminated Composites Through Multiscale Modeling of Progressive Failure," proceedings of the 48th AIAA/ASME/ASCE/AHS SDM Conference, April 22-25, 2007, Honolulu, Hawaii.

P. Hajela and V. Sakalkar, "Uncertainty Modeling in Composite Structural Design," presented at the 7th ASMO UK Engineering Design Optimization Conference, Bath, UK, July 2, 2008.

V. Sakalkar and P. Hajela, "Nondeterministic decomposition based structural design optimization", proceedings of the 49th AIAA/ASME/ASCE/AHS SDM Conference, Schaumburg, Illinois, 2008.

P. Hajela, A. Mullur, and V. Sakalkar, "Multidisciplinary Analysis and Design: Tools for Uncertainty Modeling, Journal of Aerospace Sciences and Technologies, Vol 61, No. 1, pp 240-251, February 2009.

V. Sakalkar and P. Hajela, "State Transition Approach to Reliability Based Design of Composite Structures", to appear in proceedings of the 50th AIAA/ASME/ASCE/AHS SDM Conference, Palm Spring, California, 2009

The complete report was too big to upload to the website and has been e-mailed to the Program Manager. Only the final section of the report that deals with new contributions for the year are included in this segment.

Section IV. System Reliability Assessment

A structural system composed of more than one structural element may also include multiple modes of failure, such as those resulting from large deflections, or from buckling, shear, and corrosion, among others. Evaluation of the reliability of such a system involves multiple limit state functions, and obtaining independent failure probability estimates for each constraint. Component-level reliabilities can be estimated using the techniques standard analytical techniques (FORM, SORM etc) or Monte Carlo simulations. The final system reliability is likely to be a function of the probabilities for the individual modes of failure, and any correlation between the different modes. The failure time of a system with two or more failure modes can be modeled with a series-system or competing risk model. Each risk is like a component in a series system. When one component fails, the system (i.e., product) fails. Each unit has a potential failure time associated with each failure mode. The observed failure time is the minimum of these individual potential failure times. Most generic system reliability techniques, such as the failure mode approach, may not be adequate for large-scale aerospace systems, where multiple engineering teams work concurrently on different aspects of the design. Instead, a dedicated approach that caters to their specific needs may need to be developed. A state transition approach to evaluate system reliability or effectiveness is proposed in this context. This approach takes into consideration the probabilities of a system transitioning from one failure state to another in a systematic manner.

The state transition method is an alternative approach for estimating risk in a system with multiple failure modes (limit states). The concept is loosely patterned on ideas developed in the military operations research community in the field of system effectiveness analysis. The approach is particularly significant in that it can compute the probability of a system existing in different states (failure modes) as a function of system operation history and failure mode relationship. This makes it possible to develop accurate risk models and "manage" the life of the system in a manner that maximizes usage and minimizes risk. In the context of composite design using TFA, each failure mode corresponds to progressive damage in the various constituent plies. Effectiveness is defined as a product of availability (**A**), the dependability (**D**), and the capability (**C**) of the system under consideration, and expressed mathematically as the following matrix product.

$$E = \{A_1 \quad A_2 \quad A_3 \quad \dots \quad A_i\} \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1j} \\ D_{21} & D_{22} & \cdots & D_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ D_{i1} & D_{i2} & \cdots & D_{ij} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_j \end{bmatrix} \tag{5}$$

In Eq. 5, '**E**' is the System effectiveness and is a measure of the extent to which a system will perform its mission; it is function of the probability to survive various states of damage, as well as the consequence of arriving in a particular state (condition). '**A**' is the availability, a measure of the system condition at the start of a mission. For the purpose of the system degradation, it is the probability that a system is put into service in various conditions ranging from new (full life), to a fully degraded state with no life. $\mathbf{A_i}$ is the probability of a system starting its mission state. In context of composites it can be the probability of a particular ply to be available at any point of time during its service life. '**D**' is the dependability indicated by a transition matrix, where elements of the matrix d_{ij} represent the probability of a system starting in state 'i' and ending in state 'j'. '**C**' is the capability, a measure of the system's ability to achieve a mission objective. For system-lifing applications, a system with full life will complete its desired objective that may involve supporting given loads or surviving applied temperature gradients.

In the context of a composite design problem, each of the failure modes like the ones described in Fig. 2 can be included in the design problem. The feasibility of this approach can be illustrated using a composite design problem involving two failure modes as follows.

Mode I: The failure strain at the point of first failure (ε^{f1} in Fig. 2) does not exceed an allowable value (0.5%).

Mode II: The failure strain at the point of second failure (ε^{f2} in Fig. 2) does not exceed an allowable value (0.75%).

The suitability of system effectiveness approach in probabilistic optimization is illustrated through a comparative study of deterministic and nondeterministic design optimization solutions as follows.

Case A: Deterministic optimization formulation - the composite design problem is first formulated as a deterministic multiobjective optimization problem that seeks to minimize the weight of the laminate and maximize the modulus of elasticity ($\bar{E}_{11}$) in the direction of unidirectional in-plane loading (11-direction from Fig. 2). The design variables are chosen as the volume fraction of the fiber material (c_r) and the ply angle ϕ in a [0, +/-ϕ, 90]$_s$ laminate. The design problem statement is explicitly stated as follows.

$$\begin{aligned}
&Min: Weight\\
&Max: \bar{E}_{11}\\
&s.t.\\
&\varepsilon^{f1} \le \varepsilon^{allow1}\\
&\varepsilon^{f2} \le \varepsilon^{allow2}\\
&0.2 \le c_r \le 0.9\\
&30 \le \phi \le 60, Integer
\end{aligned} \tag{6}$$

This is a design problem with both continuous and integer design variables. A genetic algorithm (GA) based optimization solution was implemented in this problem given the mixed nature of the design variables and expected multimodalities in the design space. The GA based approach required a very large number of function evaluations and the use of the RBF approximation was critical to this implementation. This formulation gives the Pareto optimal set of solutions shown in Fig. 7, and which will be used for comparison with nondeterministic cases.

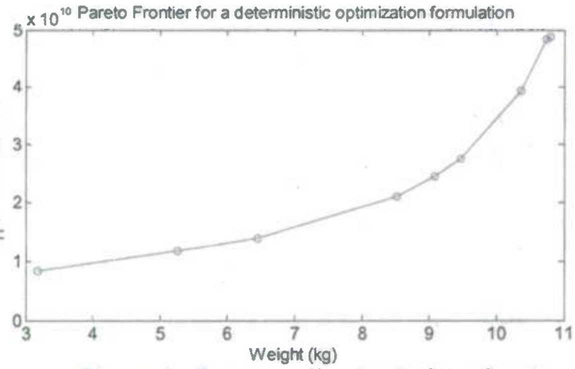

Figure 7. Pareto optimal solutions for the

Case B: Nondeterministic optimization formulation (reliability constraint) - the composite optimization problem is formulated as a non deterministic multiobjective optimization problem seeking to minimize the mean weight ($Weight^{\mu}$) of the laminate while also maximizing the mean axial modulus of elasticity ($\bar{E}_{11}^{\mu}$) of the laminate. The design variables are the volume fraction of the fiber material and the ply angle ϕ in a [0, +/-ϕ, 90]s laminate. The design problem statement is explicitly stated as follows.

$$\begin{aligned}
&Min : Weight^{\mu} \\
&Max : \bar{E}_{11}^{\mu} \\
&s.t. \\
&R_{system} \geq R_{\min};\ \ where\ \ R_{system} = (1-P_I)(1-P_{II}) = (1-P_f(\tilde{\varepsilon}^{f1} \leq \tilde{\varepsilon}^{allow1}))(1-P_f(\tilde{\varepsilon}^{f2} \leq \tilde{\varepsilon}^{allow2})) \\
&0.2 \leq \tilde{c}_r^{\mu} \leq 0.9 \\
&30 \leq \phi \leq 60, Integer
\end{aligned} \tag{7}$$

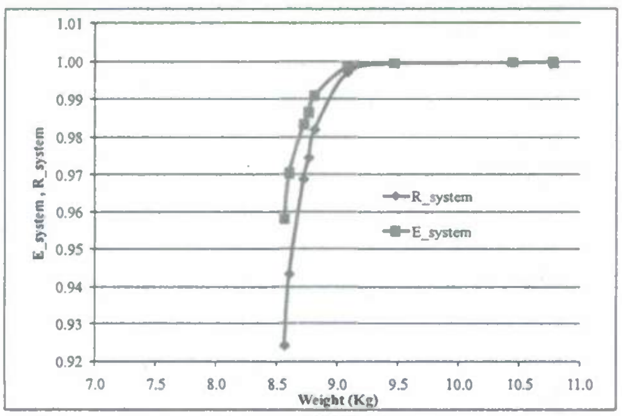

Figure 8. Pareto solution for Case B formulation

Here, $\tilde{c}_r$ is a normal random variable $\tilde{c}_r = N(\tilde{c}_r^{\mu}, 0.01)$ with its mean value ($\tilde{c}_r^{\mu}$) as the design variable in the optimization problem. The allowable failure strains follow a normal distribution, with values as $\tilde{\varepsilon}^{allow1} = N(0.5, 0.05); \tilde{\varepsilon}^{allow2} = N(0.75, 0.075)$. The system reliability (using Eq. 19) and system effectiveness (using Eq. 15) are computed for each of the solutions on the Pareto front. The results obtained are presented in Table 2 and also shown on Fig.8.

Table 2 Pareto solution for Case B formulation

Volume Fraction	Ply angle	Weight	Laminate Axial Stiffness	R_{system}	E_{system}
0.898	57	10.780	4.979E+10	1.000	1.000
0.898	56	10.775	4.949E+10	1.000	1.000
0.870	57	10.445	4.114E+10	1.000	1.000
0.789	57	9.465	2.766E+10	1.000	1.000
0.757	56	9.081	2.456E+10	0.997	0.999
0.734	56	8.811	2.278E+10	0.982	0.991
0.730	55	8.763	2.248E+10	0.975	0.986
0.727	57	8.720	2.224E+10	0.969	0.983
0.717	57	8.604	2.158E+10	0.943	0.970
0.714	55	8.563	2.135E+10	0.924	0.958

Case C: Nondeterministic optimization formulation (effectiveness constraint) – this case involves the solution of a multiobjective optimization problem as before, with the addition of system effectiveness based constraints. The problem formulation is as follows.

$$\begin{aligned}
&Min : Weight^{\mu} \\
&Max : \overline{E}_{11}^{\mu} \\
&s.t. \\
&E_{system} \geq E_{\min} \\
&0.2 \leq \tilde{c}_{r}^{\mu} \leq 0.9 \\
&30 \leq \phi \leq 60, Integer
\end{aligned} \tag{8}$$

This problem formulation is the same as in case (B) except that reliability constraint is replaced by effectiveness constraint. The results for this problem formulation are summarized in Table 3 and depicted graphically in Fig. 9.

Table 3 Pareto solutions for Case C formulation

Volume Fraction	Ply angle	Weight	Laminate Axial Stiffness	R_{system}	E_{system}
0.899	56	10.784	4.977E+10	1.000	1.000
0.896	56	10.755	4.886E+10	1.000	1.000
0.880	55	10.563	4.358E+10	1.000	1.000
0.863	56	10.354	3.925E+10	1.000	1.000
0.777	57	9.330	2.648E+10	0.999	0.999
0.730	57	8.763	2.250E+10	0.975	0.987
0.728	57	8.741	2.237E+10	0.972	0.985
0.715	57	8.584	2.147E+10	0.938	0.967
0.710	56	8.517	2.111E+10	0.915	0.956
0.695	57	8.337	2.021E+10	0.825	0.909

4.1 Discussion

In Tables 2 and 3, it is evident that the reliability is bounded between 0.825 and 1.0, and effectiveness is bounded between 0.909 and 1.0. For the same design on the Pareto front, the reliability is always lower than effectiveness. This is due to the fact that competing risk model emphasizes both failure modes equally whereas the state transition approach places less emphasis on the first failure mode. Thus, the effectiveness metric provides a more realistic estimate of the true system capability and is based on the acceptance by the designer that the residual load carrying capacity of the structure continues to be significant. This is also demonstrated in the second formulation with the effectiveness constraint. The designs generated in Tables 2 and 3 are in general lighter than the designs generated by the reliability based approach. In this case study, first failure strain was considered a benign failure, i.e., from operational perspective a laminate with initial strain failure has still completed a part of its mission. This "risk reduction" is captured by the effectiveness-based formulation and is finally reflected in the optimal designs produced.